# Failure Accommodation in Digital Flight Control Systems Accounting for Nonlinear Aircraft Dynamics

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**This paper presents a design method for optimal redundancy management for nonlinear systems with application to highly maneuvering aircraft. The approach taken is based on selecting the failure states to be covered by the system design and constructing a cost function that represents the cost of making an incorrect decision. The decision logic that minimizes the cost requires a bank of extended Kalman filters running in parallel. This produces a severe computational requirement. To reduce this requirement, a suboptimal logic is developed based on using a nonlinear single-stage prediction algorithm in the filters with filter gains and decision logic selected using steady-state results obtained from a linearization of the vehicle and sensor dynamics. The design process then is applied to designing a redundancy management system for the F8-C aircraft. The performance of the system is indicated with results of a nonlinear, piloted, six-degree-of-freedom simulation of the F8-C aircraft. Results indicate that the system is superior in failure detection to a system using the same structure but using a linear single-stage prediction algorithm in the filters.**

## Introduction

ONE important potential for digital control systems is their ability to reorganize themselves following failures in sensors and actuators. For sensors, this may be accomplished using duplication of components with simple voting to determine failures. Redundancy provided in this way can be termed "hardware" redundancy, since it owes its existence to hardware duplication. Redundancy also may exist between dissimilar components. To use this type of redundancy, one must be aware of the dynamic behavior of the system and its components. Such redundancy is called "analytical" redundancy, since it owes its existence to analytical knowledge of the system. This paper presents a design method for optimal management of analytical redundancy in nonlinear systems.

The level of redundancy that exists in a linear system has been studied in Ref. 1. In Ref. 1, conditions for detecting failures in linear systems are derived, along with a failure detection filter that is based on a prescribed failure mode model. No consideration of noise was made in Ref. 1. Later, Ref. 2 presented a method for managing redundancy in linear systems that are subject to both process noise and measurement noise. That approach was basically different from the failure detection filter approach and did not require modeling of the failure mode. Instead, multiple hypothesis testing was used to determine the most likely failure state of the system. Both Refs. 1 and 2 were limited to linear systems.

For highly maneuvering aircraft, it is necessary to consider nonlinearities, since the outputs of sensors are coupled strongly to the kinematic orientation of the vehicle. This paper parallels the theoretical developments of Ref. 2 for nonlinear systems. The approach taken is based on selecting the failure states to be included in the system design and constructing a cost function that indicates the cost of making an error in the redundancy management logic. Minimizing the cost function results in an optimal decision logic that uses a bank of extended Kalman filters[3] running in parallel. Because of the computational requirements of the optimal system, a suboptimal logic is developed based on using a nonlinear single-stage prediction algorithm using filter gains and selection logic determined from a linearization of the vehicle and sensor dynamics about a level flight condition. The unique feature of this method is that the single-stage prediction is made as accurately as possible by using a nonlinear model of the aircraft and sensor dynamics. The design process then is applied to designing a redundancy management system for the F8-C aircraft. The performance of this system is indicated using a nonlinear simulation of the F8-C airplane.

## Sensor Redundancy Management in Nonlinear Systems

Consider the equations of motion of an aircraft to be represented by the nonlinear differential equation

$$\dot{x}=f(x,u) \tag{1}$$

where $x$ is an $n$-dimensional state vector and $u$ is an $m$-dimensional control vector. For aircraft, the function $f(x,u)$ is a continuous vector function that is differentiable in $x$ and $u$. For the purpose of designing a redundancy management system controlled by a digital computer, a difference equation representation of the aircraft dynamics is required. It can be considered to be of the form

$$x(k+1)=\Phi[x(k),\ u(k)]$$

where $x(k)\triangleq x(k\tau)$, $u(t)$ in Eq. (1) is equal to $u(k)$ for $k\tau\leq t \leq (k+)\tau$, and $\tau$ is the cycle time for the digital control system. Since the system is nonlinear, an error will exist in the computation of $x(k+1)$, given $x(k)$ and $u(k)$, which depends on the form of $f(x,u)$, the method used to generate $\Phi[x(k), u(k)]$, and the time interval $\tau$. For the design process we shall, hence, consider the dynamics to be of the form

$$x(k+1)=\Phi[x(k),u(k)]+w(k)$$

where $w(k)$ is a random variable that must be added to $\Phi[x(k),\ u(k)]$ to account for errors in integration of $f(x,u)$ over the interval $\tau$. We shall consider $w(k)$ to be a stationary gaussian random process with zero mean and covariance matrix $W$ so that $E[w(k)w^T(j)]=W\delta_{kj}$. In actual aircraft operation, the process $w(k)$ is generated from several mechanisms. One is error obtained because of lack of knowledge about aircraft dynamics. Another is error obtained because of unmodeled external inputs to the vehicle (that is, atmospheric turbulence). These two factors were considered for linear systems in Ref. 2. For nonlinear systems, however,

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another error source exists because of the conversion from the continuous system representation to a difference equation representation. All factors mentioned need to be considered in selecting the statistics of the sequence $w(k)$ to be used in the design of the redundancy management logic.

Sensors $y$ will be considered to measure the vector function

$$y(k)=h^0[x(k),\ u(k)]+v^0(k)$$

for the unfailed state $H_0$. The sequence $v^0(k)$ represents the measurement error and is considered to be a zero mean gaussian process for the purpose of design. For the failure states $H_i$, $1\leq i\leq M-1$, it is assumed that the sensors' outputs are the same as those for the unfailed states except for $y_i$, whose output is a gaussian process with unknown mean. Hence, for each of the $M-1$ failure states $H_i$ the sensor outputs are

$$H_i\!:y(k)=h^i[x(k),\ u(k)]+v^i(k),\ 1\leq i\leq M-1$$

where

$$E[v^i(k)]=m_i$$
$$E[v^i(k)v^{i^T}(j)]=V^i\delta_{kj}$$

The quantity $m_i$ is an unknown parameter vector representing the mean of $v^i(k)$. It is also convenient to introduce a dummy hyptothesis $H_M$ with a priori probability $P_{H_M}=0$ with $H_M$: $y(k)=v^0(k)$.

We shall be concerned with selecting the most probable failure state $H_i$ based on a finite set of measurements $Y(k)=[y(1), y(2),\ldots, y(k)]$. To do this, a cost function is constructed of the form

$$\beta=\sum_{i=0}^{M}\sum_{j=0}^{M}P_{H_i}C_{ij}\int_{Z_i}P_{Y|H}(\alpha|H_i)\mathrm{d}\alpha \qquad (2)$$

subject to

$$\sum_{i=0}^{M}P_{H_i}=1$$

and where the sets $Z_i$, $0\leq i\leq m$, are disjoint and their union represents the entire observation space. In Eq. (2), $P_{H_i}$ is the a priori probability of the hypothesis $H_i$ occurring, $C_{ij}$ is an assigned cost of selection of $H_j$ when $H_i$ occurs, and $P_{Y|H}(\alpha|H_i)$ is the conditional probability density of the measurement sequence $\alpha$ given that $H_j$ occurs. The symbol $\int_{Z_i}$ implies that the integral is carried out over the set $Z_i$ in the observation space. The problem here is to select boundaries of the sets $Z_i$ that represent decision regions in the observation space so that when $Y\epsilon Z_i$ we will assume that $H_i$ is true. Of course, there is a possibility that an error might exist in the decision process. That is, $Y\epsilon Z_i$, but $H_j$ is true. The function $\beta$ is selected so that it represents the cost of making an incorrect decision. This function will then be minimized analytically by selecting the zones $Z_i$. The terms in Eq.(2) may be interpreted in the following way. Each of the integral terms represents the probability that a measurement sequence is in $Z_i$ and hypothesis $H_j$ is true. Hence, each integral term represents the probability of selecting $H_i$ when $H_j$ is true. These terms are multiplied by the a priori probability of $H_j$ occurring, $P_{H_j}$, and design weighting constants $C_{ij}$ to obtain the total cost function referred to in Ref. 2 as the Bayesian risk function. Therefore, the function $\beta$ represents the cost of assigning any decision structure, $Z_i$. We shall assume that the cost weights $C_{ij}$ assigned for making a correct decision are zero ($C_{ii}=0$, $i=0$, M) and that the cost of making an incorrect decision is positive ($C_{ij}>0$, $i\neq j$).

The problem, then, is to select boundaries of $Z_i$ which will result in minimum cost. The general solution to the problem is represented most easily in terms of likelihood ratios $\Lambda_i$, where

$$\Lambda_i(\alpha)=P_{Y|H}(\alpha|H_i)/P_{Y|H}(\alpha|H_M),\ i=0,\ \ldots,\ M-1$$

The cost function $\beta$ of Eq. (2) may be rewritten as

$$\beta=\sum_{i=0}^{M}\int_{Z_i}\psi_i(\alpha)\mathrm{d}\alpha$$

where

$$\psi_i(\alpha)=\sum_{j=0}^{M}P_{H_j}C_{ij}P_{Y|H}(\mathrm{A}|H_j)$$

This function is minimized by selecting $i$ at each point $\alpha$ in the observation space such that $\psi_i(\alpha)$ is the smallest of the $M+1$ possible values $\psi_k(\alpha)$, $0\leq k\leq M$. Hence

$$Z_i=\{\alpha|\psi_i(\alpha)=\min_{0\leq k\leq M}\psi_k(\alpha)\}$$

Dividing each $\psi_i$ by the probability density of $Y(k)$ under $H_M$ gives an equivalent test term, $\tau_i(\alpha)$, for making the decision in terms of the likelihood ratios:

$$\tau_i(\alpha)=\sum_{j=0}^{M-1}P_{Hj}C_{ij}\Lambda_j(\alpha)$$

Then

$$Z_i=\{\alpha|\tau_i(\alpha)=\min_{0\leq k\leq M-1}\tau_k(\alpha)\}$$

Consider now the problem of mechanizing the preceding test. That is the problem of determining the likelihood ratios $\Lambda_i(\alpha)$ for the hypothesis

$$H_i\!:y(k)=h^i[x(k)]\ +\ v^i(k),\ i=0,\ 1,\ \ldots,\ M-1,\quad k=0,\ 1,\ \ldots,\ K$$

By the chain rule for probability densities,

$$p[Y(k)|H_i]=p[(y)|Y(k-1),\ H_i]\ p[Y(k-1)|H_i]$$
$$=p[y(1)|H_i]\prod_{j=2}^{k}p[y(j)|Y(j-1),\ H_i]$$

Since under hypothesis $H_i$, $y(j)=h^i[x(j)]+v^i(j)$ and $h$ is nonlinear, the probability density $p[y(j)|Y(j-1),\ H_i]$ is nongaussian. However, the first two moments can be expressed by

$$E\{y(j)|Y(j-1),\ H_i\}=\hat{h}^i[x(j)]+m_i$$

where

$$\hat{h}^i[x(j)]=E\{h^i[x(j)]|Y(j-1),\ H_i\}$$

and $m_i=E\{v^i\}$; and

$$\mathrm{var}\{y(j)|Y(j-1),\ H_i\}=V_{\hat{h}}i(j|j-1)\ +\ V_v^i$$

where

$$V_{\hat{h}}i(j|j-1)=E\{[h^i(x(j)) -\hat{h}^i(x(j))]\ [h^i(x(j)) -\hat{h}^i(x(j))]^T|Y(j-1),\ h_i\}$$

If it is assumed that the density $p[y(j)|y(j-1),H_i]$ is gaussian, then the first two moments give the pseudo-Bayes density function

$$p[y(j)|Y(j-1),H_i]=\{(2\pi)^N\det[V_{\hat{h}}i(j|j-1)\ V_v^i]\}^{-1/2}$$
$$\times\ \exp\{-{1/2}[y(j)-\hat{h}^i(x(j))-m_i]^T[V_{\hat{h}}i(j|j-1)$$
$$+\ V_v^i]^{-1}[y(j)-\hat{h}^i(x(j)-m_i]\}$$

where $N=\dim y$. McLendon[4] shows that, when the variance $V_{\hat{h}}i\ (j|j-1)$ is small compared to $V_v^i$, the pseudo-Bayes assumption can be justified analytically.

Then the likelihood ratio becomes, after some simplification,

$$\Lambda_i(K) = \prod_{j=1}^{K} \frac{\{\det(V_v^0)\}^{1/2}}{\{\det[V_h^i(j|j-1) + V_v^i]\}^{1/2}}$$
$$\times \exp\{ \frac{1}{2} \sum_{j=1}^{K} [y(j)^T V_v^{0^{-1}} y(j)$$
$$-\{y(j) - h\hat{h}^i[x(j)] - m_i\}^T \times [V_{\hat{h}} i(j|j-1)$$
$$+ V_v^i]^{-1}\{y(j) - \hat{h}^i[x(j)] - m_i\}]\}$$

In order to calculate this likelihood ratio, one must have values for $\hat{h}^i[x(j)]$ and $V_{\hat{h}}(j|j-1)$. Since $h$ is nonlinear, these cannot be computed exactly; however, expanding $h^i[x(j)]$ in a Taylor series to first order about $\hat{x}^i(j|j-1) \triangleq E\{x(j)\,|\,Y(j-1), H_i\}$ gives $\hat{h}^i[x(j)] = h[\hat{x}^i(j|j-1)]$ and

$$V_h(j|j-1) = (\partial h^i/\partial x)[\hat{x}^i(j|j-1)]\, V_x i(j|j-1)$$
$$\times (\partial h^{i^T}/\partial x)[\hat{x}^i(j|j-1)]$$

In Ref. 2 for linear systems $\hat{x}^i(j|j-1)$ was obtained using a linear discrete Kalman filter. For the nonlinear problem, however, a nonlinear estimator is required. The result can be obtained only approximately for the nonlinear case using an approximate conditional-mean filter.[3] The filter algorithm is derived in Ref. 3 and is presented here for completeness. Consider $H_i$, and define

$$\hat{x}^i(k+1) \triangleq E[x(k+1)\,|\,Y(k+1), H_i]$$
$$\hat{x}^i(k+1|k) \triangleq \Phi[\hat{x}^i(k), u(k)]$$

and

$$r^i(k) \triangleq y(k) - h[\hat{x}(k), u(k)] - \tilde{m}_i(k)$$

where $\tilde{m}_i(k)$ is the maximum likelihood estimate of $m_i$ over the observation sequence. With these definitions, the filter algorithm is

$$\hat{x}^i(k+1) = \hat{x}^i(k+1|k) + K^i(k+1)r^i(k+1)$$

The gain matrix $K^i(k)$ can be calculated using

$$K^i(k+1) = V_{\tilde{x}}^i(k+1|k)\,(\partial h^{i^T}/\partial x)[\hat{x}^i(k+1|k),$$
$$u(k+1)]\, V_y^{i^{-1}}(k+1)$$
$$V_y^i(k+1) = (\partial h^i/\partial x)[\hat{x}^i(k+1|k), u(k+1)]$$
$$V_{\tilde{x}}^i(k+1|k)(\partial h^{i^T}/\partial x)[\hat{x}^i(k+1|k), u(k+1)]$$
$$\times u(k+1) + V_v^i$$

where $V_{\tilde{x}}^i(k+1|k)$ is the prediction error covariance given by

$$V_{\tilde{x}}^i(k+1|k) = (\partial\Phi/\partial x)[\hat{x}^i(k), u(k)]\, V_{\tilde{x}}^i(k)$$
$$\times (\partial\Phi^T/\partial x)\;[\hat{x}^i(k), u(k)] + W$$

and $V_{\tilde{x}}^i(k)$ is the filter error covariance satisfying

$$V_{\tilde{x}}^i(k+1) = \{I - K^i(k+1)(\partial h/\partial x)[\hat{x}^i(k+1),$$
$$u(k)]\}\; V_{\tilde{x}}^i(k+1|k)$$

Hence, for each $H_i$ there is a filter algorithm, as indicated previously, required to accomplish the decision process. The structure of the optimal decision logic that might be implemented in a digital flight control system is indicated in Fig. 1. To simplify the decision process, we may take $C_{ij} = 1$, $i \neq j$. In this case the optimal decision process involves selecting for each measurement sequence $Y(K)$ the hypothesis corresponding to maximizing

$$\ln P_{Hi} - \ln \prod_{j=1}^{K} |V^i(j)| - \frac{1}{2} \sum_{j=1}^{K} r^{i^T}(j) V^{i^{-1}}(j) r^i(j)$$

where $K$ is the total number of measurements used to make the decision. The preceding conditional mean filtering algorithm was coded for an example aircraft problem on the Langley Research Center real-time computing system using a CDC 6600 computer. It was found that to implement a single filter in the manner just described required that the computer run two-to-one slow. Thus, the computational load required for a system of the type indicated in Fig. 1 is too severe to make it practical for implementation. Most of the computational burden of the systems stems from updating the covariance matrices required by the filter. A suboptimal filter was, therefore, selected for the application section of this paper which does not require updating of the covariance matrices. It involves a filter using constant gains and constant weights of the residual sequence $r^i(k)$. However, the filter does use a nonlinear single-st ge prediction to obtain $\hat{x}^i(k+1|k)$. The filter gains were steady-state gains obtained from a linearization of the aircraft dynamics about the level flight condition. Under these assumptions, the preceding algorithm holds except that $\partial\Phi/\partial x$ and $\partial h/\partial x$ are held constant. The use of steady-state filter gains further reduces the computational load for the decision process to selecting the hypothesis corresponding to minimizing

$$\frac{K}{2} \ln |V_y^i| + \frac{1}{2} \sum_{j=1}^{K} r^{i^T}(j) V_v^{i^{-1}} r^i(j) \qquad (3)$$

where $V_y^i = \lim k \to \infty V_y^i(k)$ in the preceding algorithm. The next section reports the application of this decision logic to the F8-C airplane using a real-time digital simulation.

## Application to Aircraft Sensor Failure Detection

The theory developed in the previous section was applied to the design of a failure detection system for the F8-C digital fly-by-wire aircraft. The system was designed to detect failures in sensors used for lateral motions. These sensors were a roll rate gyro, a yaw rate gyro, and a sideslip angle indicator. Sensors for longitudinal motions were used, but the system was not designed to recover from failures of those sensors. The longitudinal sensors were total airspeed, angle of attack, and pitch rate. This section is devoted to items that need to be considered in designing an analytical redundancy management system and results obtained from a real-time, piloted, six-degree-of-freedom simulation of the F8-C on which the system was tested. That simulation used a complete nonlinear set of aerodynamic data. It is described in Ref. 5.

One important item that must be considered in the design of a failure detection system for aircraft is the model used by the onboard digital computer to represent the dynamics of the vehicle. Of concern is the accuracy and complexity of the model. In selection of the model of the aircraft dynamics used here, it was considered that the aircraft's stability derivatives were not sufficiently well known to justify the scheduling of such data as functions of flight condition and configuration changes. Also, it was felt that the representation of the aircraft's kinematic equations was well understood and could be scheduled more conveniently as a function of aircraft con-

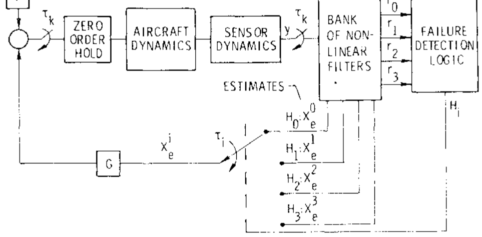


**Fig. 1 Analytical redundancy management scheme block diagram.**

figuration. Therefore, in an effort to use as simple a model as possible and still provide an accurate representation of the aircraft dynamics, the model selected was one in which linear aerodynamics were assumed but the kinematic relations were included as nonlinear functions. The term "linear aerodynamics" is meant to imply that the aerodynamic force and moment coefficients, $X, Y, Z, L, M, N$, are assumed to be linear functions of estimated angle of attack, sideslip angle,

Again, this is a source of error in modeling of the aircraft dynamics and should be included in assigning the statistics of $w(k)$.

In designing the redundancy management logic, the gradients $\partial\Phi/\partial x$ and $\partial h/\partial x$ were held constant, and steady-state results were used in the bank of filters (Fig. 1) and in the decision functions. The covariance $W$ of $w(k)$ was taken as

$$W=\begin{bmatrix} 1.273(10)^{-4} & 3.964(10)^{-6} & -2.444(10)^{-8} & 2.420(10)^{-8} \\ 3.964(10)^{-6} & 1.740(10)^{-7} & -1.543(10)^{-9} & 1.112(10)^{-9} \\ -2.444(10)^{-8} & -1.543(10)^{-9} & 6.078(10)^{-6} & -1.693(10)^{-7} \\ 2.420(10)^{-8} & 1.112(10)^{-9} & -1.693(10)^{-7} & 9.963(10)^{-8} \end{bmatrix}$$

angular velocities, and control positions. The model selected is indicated below

$$\dot{u} = rv - qw + X/m - g\sin\theta + T\cos\theta_T$$
$$\dot{v} = pw - ru + Y/m + g\cos\theta\sin\Phi$$
$$\dot{w} = qu - pv + Z/m + g\cos\theta\cos\Phi + T\sin\theta_T$$
$$\dot{p} = [(I_y - I_z)qr + L]/I_x$$
$$\dot{q} = [(I_z - I_x)pr + M + Tl_T]/I_y$$
$$\dot{r} = [(I_x - I_y)pq + N]/I_z$$
$$\dot{\theta} = q\cos\Phi - r\sin\Phi$$
$$\dot{\phi} = p + (q\sin\theta\sin\Phi + r\sin\theta\cos\Phi)/\cos\theta$$

and the aerodynamic forces and moments are given by

$$V = (u^2 + v^2 + w^2)^{1/2}$$
$$\alpha = \tan^{-1} w/u$$
$$\beta = \sin^{-1} v/V$$
$$(X,Y,Z) = 1/2\ \rho V^2 S(C_x, C_y, C_z)$$
$$(L,N) = 1/2\ \rho V^2 Sb(C_l, C_n)$$
$$M = 1/2\ \rho V^2 S\bar{c}\, C_m$$

The representation of aerodynamic force coefficients was selected as

$$C_x = C_{x_o} + C_{x_\alpha} + C_{x_{\delta_f}}\delta_f + C_{x_{\delta_e}}\delta_e$$
$$C_y = C_{y_\beta}\beta + C_{y_{\delta_r}}\delta_r$$
$$C_z = C_{z_0} + C_{z_\alpha}\alpha + C_{z_{\delta_f}}\delta_f + C_{z_{\delta_e}}\delta_e$$
$$C_l = C_{l_\beta}\beta + C_{l_p}(pb/2V) + C_{l_r}(rb/2V) + C_{l_{\delta a}}\delta a + C_{l_{\delta_r}}\delta_r$$
$$C_m = C_{m_o} + C_{m_\alpha}\alpha + C_{m_q}(\bar{q}c/2V) + C_{m_{\delta_e}}\delta e + C_{m_{\delta_f}}\delta f$$
$$C_n = C_{n_\beta}\beta + C_{n_p}(pb/2V) + C_{n_r}(rb/2V) + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r$$

The specific data used in the model were taken from a linearization of the F8-C aerodynamics at 20,000-ft alt and 0.6 Mach number. The data are listed in Table 1, along with geometric and configuration data for the model. Note that the model assumes principal axes, whereas the sensors are body axis mounted. This introduces a modeling error that should be accounted for in the selection of the statistics of the process error $w(k)$. The preceding equations represent the continuous-time process model. The use of this model in a digital control system requires a discrete model representation. Euler's method was used to generate the discrete model difference equations using a time interval of $\tau = 0.03125$ sec. Hence, for example, the preceding roll-rate equation becomes

$$p[(K+1)\tau] = p(k\tau) + \tau\dot{p}(k\tau)$$

It was computed using the techniques of Ref. 2.

The unfailed noise level for the sensors corresponded to a measurement error with $V_v^\circ = \text{diag}\,(0.001, 0.001, 0.001)$. Table 2 lists formulas for the decision function and constants obtained using a failure mode measurement error, where

$$V_v^1 = \text{diag}\,(0.01, 0.001, 0.001)$$
$$V_v^2 = \text{diag}\,(0.001, 0.01, 0.0001)$$
$$V_v^3 = \text{diag}\,(0.0001, 0.0001, 0.01)$$

Filter gains also are listed for each hypothesis testing filter. The single-stage prediction $\hat{x}^i(k+1\,|k)$ for each filter is obtained by using the last state estimate $\hat{x}^i(k\,|k)$ of that filter and the control surface positions and computing the prediction $\hat{x}^i(k+1\,|k)$ from the nonlinear model just described. To get an appreciation for the improvement obtained using a nonlinear model, Fig. 2 indicates the performance of the failure detection logic for a system using the same detection logic but using a linearization of the aircraft's dynamics as developed in Ref. 2. In the figures $H$ represents the actual failure state of the system, whereas $\hat{H}$ represents the onboard computers' estimate of the failure state. The aircraft is subject to an aileron input causing a roll, which is seen by the system as a failure in the sensors. This occurs because the modeling error increases in the aircraft rolls to a bank angle different from the one used in the linearization. In this case, the failure detection logic produces erratic results even when no failures have occurred. The erratic behavior of the state estimate is caused by excessive model error. Figure 3 shows the behavior of the same decision logic using the nonlinear single-stage prediction formulas. In this case, the decision process is much more stable.

Figure 4 indicates the detection capability of the system for consecutive failures and recoveries of each of the three lateral sensors in the increased noise model with no maneuvers. Note

**Table 1 Constants used for the single stage prediction model**

| | | |
|---|---|---|
| Mass = 700 slugs | $I_X = 10{,}068.5$ slug-ft$^2$ | |
| $g = 32.17$ fps$^2$ | $I_Y = 87{,}500.$ slug-ft$^2$ | |
| $\theta_T = 0$ | $I_Z = 93{,}101.4$ slug-ft$^2$ | |
| $l_T = 0$ | $\rho = 0.00127$ slug/ft$^2$ | |
| $b = 35.7$ ft | $S = 375.0$ ft$^2$ | |
| $\bar{c} = 11.75$ ft | | |
| $C_{X_0} = -0.0202$ | $C_{Y_\beta} = -0.88$ | $C_{Z_0} = -0.173$ |
| $C_{X_\alpha} = -0.0784$ | $C_{Y_{\delta_r}} = 0.2$ | $C_{Z_\alpha} = -4.127$ |
| $C_{X_{\delta_f}} = 0$ | | $C_{Z_{\delta_f}} = -0.6415$ |
| $C_{X_{\delta_l}} = 0$ | | $C_{Z_{\delta_l}} = 0$ |
| $C_{l_\beta} = -0.1164$ | $C_{m_0} = 0.001$ | $C_{n_\beta} = 0.197$ |
| $C_{l_p}(b/2) = -6.237$ | $C_{m_\alpha} = -0.307$ | $C_n(b/2) = -0.245$ |
| $C_{l_r}(b/2) = 0.09736$ | $C_{m_q}(\bar{c}/2) = -21.338$ | $C_{n_r}(b/2) = -5.73$ |
| $C_{l_{\delta_a}} = 0.05791$ | $C_{m_{\delta_l}} = 0.866$ | $C_{n_{\delta_a}} = 0.00736$ |
| $C_{l_{\delta_r}} = 0.0266$ | $C_{m_{\delta_f}} = 0$ | $C_{n_{\delta_r}} = -0.1078$ |

**Table 2 Failure detection logic parameters for a five-sample data base**

$H_0$

$$K^0(\infty) = \begin{bmatrix} 6.419(10)^{-1} & 6.130(10)^{-4} & 1.449(10)^{-4} \\ 2.998(10)^{-2} & 8.285(10)^{-4} & 1.275(10)^{-3} \\ 6.130(10)^{-4} & 2.070(10)^{-1} & -5.802(10)^{-3} \\ 1.449(10)^{-4} & -5.802(10)^{-3} & 2.930(10)^{-2} \end{bmatrix}$$

$$V_y^{0^{-1}} = \begin{bmatrix} 3.578(10)^3 & -1.834(10) & -3.256 \\ -1.834(10) & 7.926(10)^3 & -2.696(10)^2 \\ -3.256 & -2.696(10)^2 & 9.656(10)^3 \end{bmatrix}$$

$$\ell n(\det V_y^0) = -2.633(10)$$

$H_1$

$$K^1(\infty) = \begin{bmatrix} 0. & 9.537(10)^{-3} & 2.790(10)^{-2} \\ 0. & 2.054(10)^{-1} & 2.677(10)^{-1} \\ 0. & 2.117(10)^{-1} & -1.261(10)^{-4} \\ 0. & -1.261(10)^{-4} & 3.634(10)^{-2} \end{bmatrix}$$

$$V_y^{1^{-1}} = \begin{bmatrix} 1.000(10)^2 & 0. & 0. \\ 0. & 7.896(10)^3 & -3.179(10)^2 \\ 0. & -3.179(10)^2 & 9.570(10)^3 \end{bmatrix}$$

$$\ell n(\det V_y^1) = -2.274(10)$$

$H_2$

$$K^2(\infty) = \begin{bmatrix} 6.419(10)^{-1} & 0. & -3.819(10)^{-4} \\ 3.000(10)^{-2} & 0. & 3.783(10)^{-4} \\ 3.599(10)^{-3} & 0. & -8.606(10)^{-2} \\ -3.819(10)^{-4} & 0. & 7.155(10)^{-2} \end{bmatrix}$$

$$V_y^{2^{-1}} = \begin{bmatrix} 3.578(10)^3 & 0. & -4.025 \\ 0. & 1.0000(10)^2 & 0. \\ -4.025 & 0. & 9.361(10)^3 \end{bmatrix}$$

$$\ell n(\det V_y^2) = -2.193(10)$$

$H_3$

$$K^3(\infty) = \begin{bmatrix} 6.419(10)^{-1} & 6.519(10)^{-4} & 0. \\ 2.999(10)^{-2} & 1.661(10)^{-3} & 0. \\ 6.519(10)^{-4} & 2.145(10)^{-1} & 0. \\ 2.006(10)^{-4} & 3.724(10)^{-3} & 0. \end{bmatrix}$$

$$V_y^{3^{-1}} = \begin{bmatrix} 3.578(10)^3 & -1.848(10) & 0. \\ -1.848(10) & 7.876(10)^3 & 0. \\ 0. & 0. & 1.000(10)^2 \end{bmatrix}$$

$$\ell n(\det V_y^3) = -2.175$$

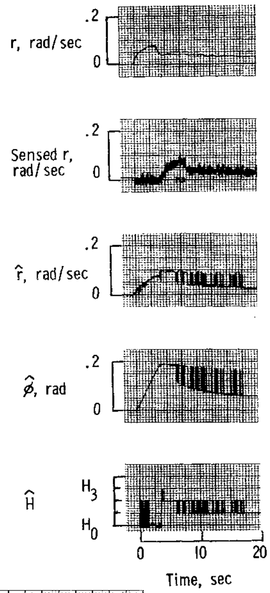


**Fig. 2 Performance of the failure detection system using a linear process model.**

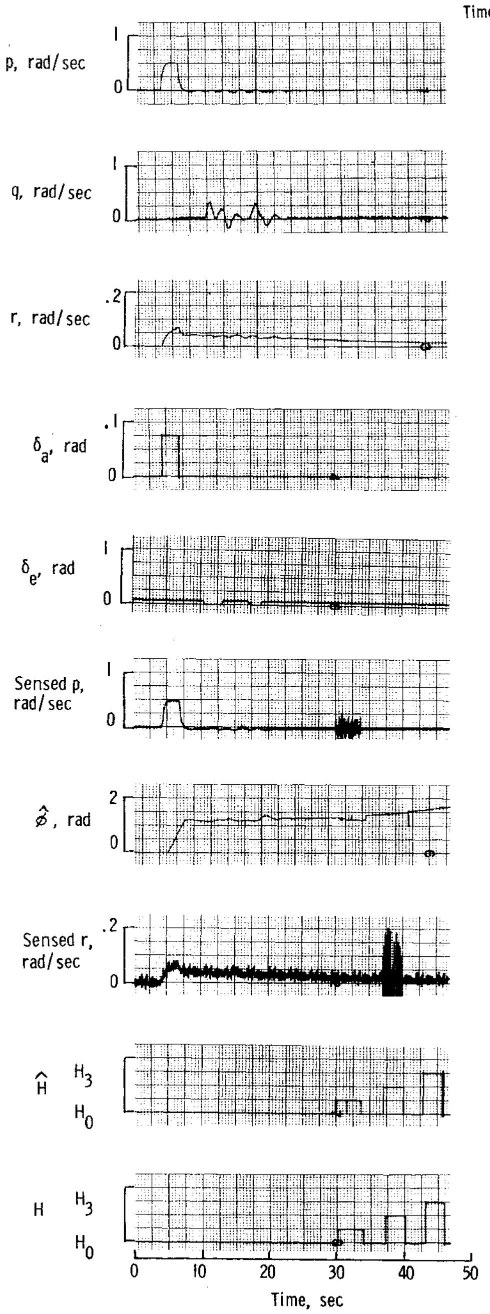


**Fig. 3 Performance of the failure detection system using a nonlinear process model.**

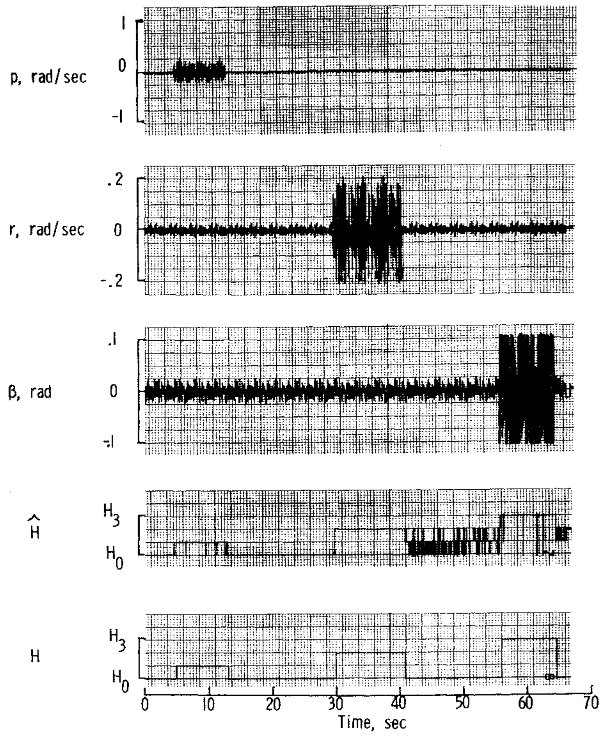


**Fig. 4 Performance of the failure detection system under increased noise mode failures with no maneuvering transients.**

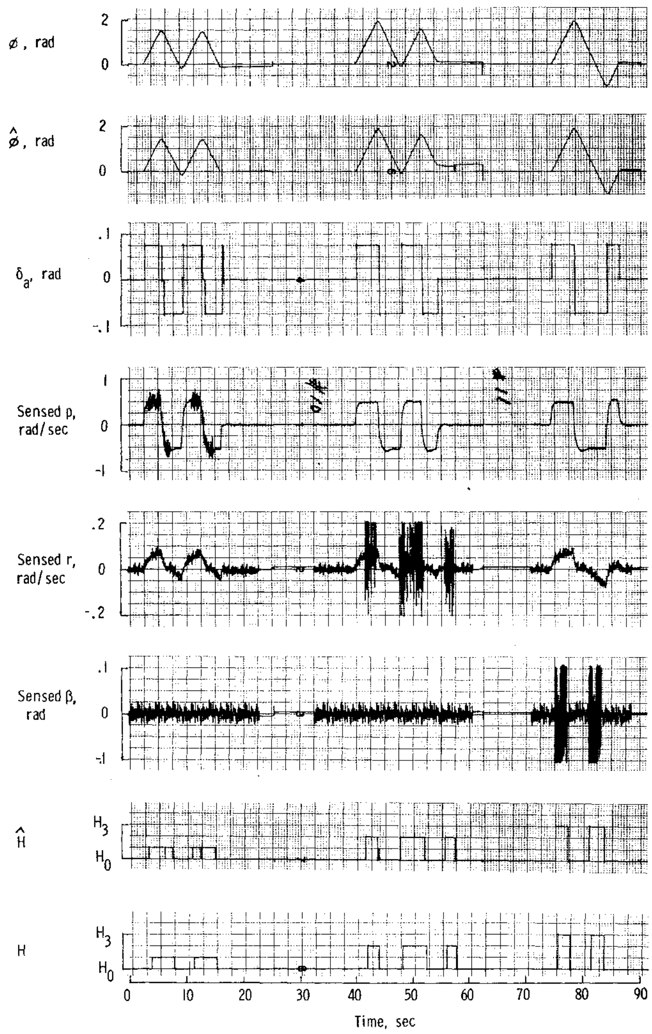


**Fig. 5 Performance of the failure detection system under increased noise mode failures with maneuvering transients introduced by aileron inputs.**

that the roll-rate gyro failure is more difficult to resolve, since the failure mode noise is not so great as it is for the other sensors. This is indicated by the switching from $H_1$ to $H_0$ when the roll-rate gyro is failed. Also note that, after corruption of the estimate of the hypothesis testing filters by a specific failure, some time may be required before the failure detection logic becomes stable again after the component failure has been removed. This is illustrated when the second sensor is failed and the failure is removed. Note that the subsequent failure $H_3$ is identified rapidly. Figure 5 indicates the performance of the decision logic for failures that occur during maneuvering transients induced by aileron inputs. The performance of the failure detection logic is not substantially different from that of the nonmaneuvering case (Fig. 4). Note that Fig. 3 also indicates system failure during maneuvering transients caused by elevator inputs while the vehicle was banked at an angle of approximately 50° to 60°.

The system also tested with hardover failures of the zero output, maximum output, and minimum output types. The detection logic behaved similarly to the tests performed under increased noise types of failures. An exception was the zero output case. In that case, the failure detection logic was not able to detect failures until the system was excited with some input, since the sensor output was supposed to be zero until some input causes an output. Failures of this type, if required to be detected during a nonmaneuvering state, would have to be handled using built-in test equipment.

## Conclusions

An analytical redundancy management system has been developed which can be used on highly maneuvering aircraft. The system has been demonstrated on a real-time, piloted simulation of the F8-C airplane. The failure detection logic used multiple hypothesis testing and depends on a pseudo-optional state estimator that uses, for each hypothesis tested, a nonlinear single-stage prediction algorithm and a decision function that used constant gains obtained from a linearization of the dynamics of the aircraft at the level flight condition. The system was tested in a real-time piloted simulation of the F8-C airplane. The tests showed that the system is superior in failure detection performance to a system using the same decision structure but using a linearization of the aircraft dynamics to generate the single-stage prediction for the state estimator. The system is being studied now to determine the knowledge required concerning vehicle dynamics to obtain satisfactory failure detection over a range of flight conditions and configuration changes.

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